



A COMPARATIVE STUDY BETWEEN GRILLAGE AND GIRDER SUPPORTING DECK STRUCTURES SUBJECTED TO A UNIFORM DECK LOADING USING FINITE ELEMENT METHOD

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Abstract— This paper presents the design and strength assessment of several grillage and girder-supported structures under uniformly distributed load to find the one that saves materials without compromising the structural integrity. For this study, the initial design idea is derived from the deck structure of an existing ship. Then it is redesigned according to the desired load applying to several design combination cases planned with various spacing between the structural members. Each of the cases is optimized at around maximum allowable stress by altering various design variables, such as girders' height and thickness, flanges' width and thickness, the thickness of deck plating, and the inclusion of stiffeners and pillars. For estimating initial design parameters and local strength analytical formulas have been used and detail Strength performance is investigated utilizing the finite element method for each case. The study explores cost-effective design options, their applications, and limitations across various scenarios.

Keywords— Stress, Deflection, Section modulus, FEA, Scantling calculation

I. NOMENCLATURE

| | |
|------|------------------------------------|
| a | long edge of a plate panel |
| b | short edge of a plate panel |
| t | thickness of a plate |
| P | designed pressure |
| M | bending moment |
| w | load per unit length |
| L, l | length of a beam or grillage panel |
| B | breadth of a grillage panel |
| Z | section modulus of a beam |

| | |
|----------------|--|
| I | moment of inertia |
| E | Young's Modulus of the material |
| D | flexural rigidity of a stiffener |
| x, y | distance from the end of a beam to the reaction point |
| F_A | axial load on a pillar |
| A_c | cross-section area of a pillar in cm^2 |
| σ | stress |
| y_{max} | deflection at the center of a plate panel |
| Δ_{max} | deflection of a beam |
| δ | deflection of a simple grillage |
| α_p | correction factor for the panel aspect ratio |
| l_s | spacing between longitudinal stiffeners (see fig. 1) |
| t_s | spacing between transverse stiffeners (see fig. 1) |
| G | spacing between Girder's stiffeners (see fig. 1) |
| S | spacing between Web Frame stiffeners (see fig. 1) |
| R_{eH} | the minimum yield stress of the material |
| χ | the coefficient taken equal to 1 |
| C_a | bending stress coefficient values 0.8 |
| f_{bdg} | bending moment distribution factor taken 24 at the mid of the span of the girder 12 at the end. taken 12 for stiffeners. |
| l_{bdg} | effective bending span of the beam |
| C_s | bending stress coefficient taken equal to 1 |
| s | spacing between stiffeners |
| S | spacing between Girders in m |
| b_{a-sup} | the mean breadth of the area supported by the pillar |
| l_{a-sup} | the mean length of the area supported by the pillar |
| w_b | height of the web of a supporting member (see Table 3) |
| flg | breadth of the flange of a supporting member (see Table 3) |
| OD | outer diameter of a pillar (see Table 3) |
| 1 _P | Case 1 with the support of pillar (see Table 3) |



- 1_{GS} Case 1 converted to girder-supported structure (see Table 3)
- 1_{GSP} Case 1 converted to girder-supported structure with pillar support (see Table 3)
- g index of the intersections of longitudinal girders
- b index of the intersections of transverse beams
- \emptyset absence of element

II. INTRODUCTION

The finite element approach for analyzing ship structure has received significant interest in recent years such as estimating the compression behavior of stiffened plates by Khosrow Ghavami (2006), investigation on the collapse behaviors of stiffened panels under combined loads by Jeom Kee Paik (2001). Bin Yang (2018) explored the impact of initial imperfections, lateral pressure, and strain rate on the ultimate strength of stiffened panels subjected to in plane dynamic compression using Finite Element Method (FEM), introducing numerical analysis in determining structural scantling by the classification societies Bureau Varitus (2024). Ming Cai Xu (2018) developed some analytical formulas for estimating the ultimate strength of stiffened panels under axial and lateral pressures through regression analysis based on FEM results. Currently, finite element analysis (FEA) is widely employed in the design of ocean-going vessels and offshore structures to ensure structural integrity under complex loading conditions, including static and dynamic sea pressures, hogging and sagging moments, dynamic cargo loads, and slamming effects. This method involves detailed mathematical modeling of real-world structures to thoroughly assess their strength. While FEA can be time-consuming and its accuracy heavily relies on the user's expertise in finite element modeling, guidelines provided by the International Association of Classification Societies (IACS) offer a framework for conducting structural analysis of ship components. Additionally, analytical formulas can estimate local structural strength by simplifying specific elements like beams or plates. The grillage analogy is applicable for analyzing stiffened plates such as decks, hull bottoms, side shells, and bulkheads. Various methods for grillage analysis, including Navier's Energy Method, the Displacement Method by Clarkson (1965), the Force Method by Wunderlich (2003), and approximate techniques like the Orthotropic Plate Method (OPM) and the Energy Method (EM) by VEDELER (1945), are available. However, their complexity, assumptions, and limitations have made them less favored compared to finite element methods.

The strength of the deck plays a prominent role for a ship for her functions like carrying cargo but it has limited space for this purpose. Most of the space gets allocated for machinery, equipment, accommodation, tanks, or other compartments especially when the ship is small in size. As a result, the decks got to carry larger loads within limited space and required to construct a deck using less steel to reduce light weight of the ship. Girder-supported decks are commonly employed in ship construction but are limited in their applicability within ship structures. In contrast, grillage structures find extensive use in designing bridge decks, buildings, and airplane structures due to their high load-bearing capacity. Therefore, there arises an interest in determining whether modifying the current design would enhance its economic viability, and exploring alternative methods to achieve greater cost efficiency in the structure.

III. METHODOLOGY

Effective analysis, design, and planning are essential for executing tasks efficiently. Initially, it is important to adhere to specific guidelines for designing and analyzing the deck structure. For this purpose, the recommended classification rules are outlined in Bureau Varitus (2024). These rules will be used to determine the minimum local structural requirements, such as plate thickness, section modulus of stiffeners, and girders, to estimate local structural strength. Additionally, numerical investigation guidelines will be followed, which include setting element types, element sizes, aspect ratios, meshing, defining loads, and post processing. To determine the unsupported span of the deck, considerations will include the ship's breadth in the transverse direction and the length between bulkheads in the longitudinal direction. Figure 1 illustrates a typical deck concept model.

To identify the most optimized structure, several design scenarios have been planned by varying the spacing of stiffeners in the girder-supported structure and the spacing of girders and webs in the grillage system, where stiffeners are absent. In the initial phase, all longitudinal girders and transverse frames will have identical cross-sections. Once the optimal configuration is determined, it will be redesigned to enhance cost-efficiency by adjusting the dimensions of the primary supporting members and incorporating reinforcements. Table 1 outlines the basic concept of these cases, where the most effective solution will be evaluated across multiple scenarios, including girder-supported structures, pillar supported structures, and combined girder and pillar support systems.

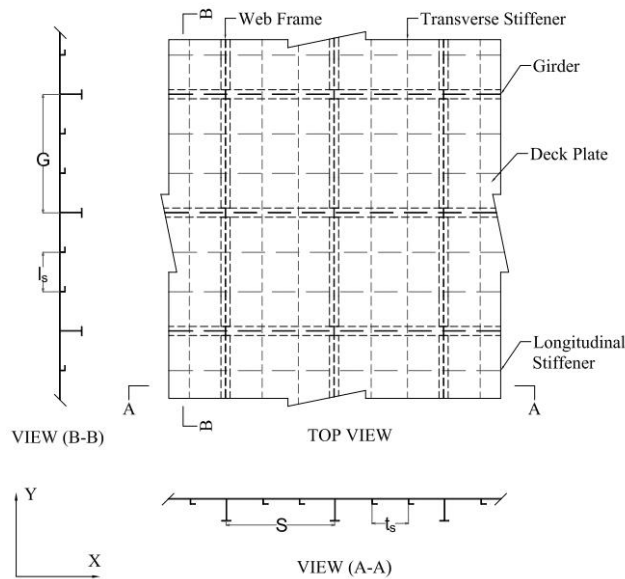


Fig. 1. Design concept of the deck structure

Subsequently, empirical formulas will be employed to estimate local strength and assess whether modifications to the girder dimensions, as determined from scantling, are necessary. Finite Element Analysis (FEA) will then be conducted to validate the accuracy of both the strength assessment and its acceptance. Any necessary design adjustments will be made throughout the

study, with this iterative process continuing until the structure demonstrates sufficient strength to support the intended load. In the final stage, the weights of all structures will be measured to select the most optimized design, which will then undergo further investigation to enhance material efficiency

Table -1 Cases for analysis (unit: mm)

| Case | l_s | t_s | G | S |
|------|-------|-------|------|------|
| 1** | 600 | 500 | 1800 | 1500 |
| 2 | Ø | 500 | 1200 | 1500 |
| 3 | Ø | Ø | 1800 | 1500 |
| 4 | Ø | Ø | 1500 | 1500 |
| 5 | Ø | Ø | 1200 | 1200 |
| 6 | Ø | Ø | 1000 | 1000 |
| 7 | Ø | Ø | 500 | 500 |

IV. STRUCTURAL MODEL AND ASSUMPTIONS

A. Analytical Approach

1) Scantling calculation

The deck structure has a length of 11 meters and a breadth of 10 meters, with a static pressure load parameter P set to 10 tons/m², representing a uniformly distributed load across the deck. Other factors such as dynamic cargo loads and weather loads on the exposed deck are assumed to be accounted for within this pressure. Since the vessel is not designed for ocean voyages, effect of wave-induced bending moments is considered to be significantly lower to affect the deck structure than the specified load due to the vessel's size (overall length: 44.6 meters, breadth: 10 meters, draft: 2.2 meters, depth: 3.5 meters). The first step to design the deck structure is find the

deck plate thickness that is calculated for the specified cases using the following formula:

$$t = 0.0158 \alpha_p b \sqrt{(|P| / (\chi C_a R_{eh}))}$$

Where,

$$\alpha_p = 1.2 - (b / (2a))$$

For plate supporting structural members it is required to determine the minimum section modulus considering the span length and spacing of the members that provide local boundary conditions and present the structure like a 2d beam beam that has fixed end both side. The following formulas are required for stiffeners, girders and web frames respectively.

$$Z = (|P| s l_{bdg}^2) / (f_{bdg} \chi C_s R_{eh})$$



$$Z_{n50} = 1000 (|P| S l_{bdg}^2) / (\chi f_{bdg} C_s R_{ch})$$

The cases are analyzed through the mentioned formula and a preliminary estimation of the design parameters are found that is shown in the Figure 2

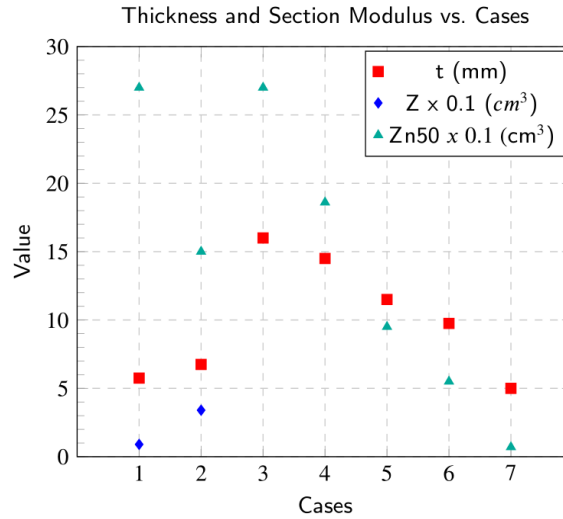


Fig. 2. Estimation of the structural scantling parameter

2) Local strength analysis

To verify the parameters obtained through the scantling calculation, several empirical formulas have been applied to both the plate and its supporting members. The initial analysis of the plate panel is conducted using the flat plate theory as presented by WARREN C. YOUNG. Specifically, the stress formula for a plate panel subjected to a uniformly distributed load across its entire surface, with all edges fixed, has been utilized.

$$\sigma_{max} \text{ (at the center of long edge)} = (-\beta_1 q b^2) / t^2$$

$$\sigma_{max} \text{ (at the center)} = (\beta_2 q b^2) / t^2$$

To determine the deflection of the plate panel, the following formula was used.

$$y_{max} \text{ (at the center)} = (\alpha q b^4) / (E t^3)$$

The corresponding values for β_1 , β_2 & α can be found in Table 2.

Table -2 Values of the coefficients for plate formula

| a/b | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | ∞ |
|-----------|--------|--------|--------|--------|--------|----------|
| β_1 | 0.3078 | 0.3834 | 0.4680 | 0.4872 | 0.4974 | 0.5000 |
| β_2 | 0.1386 | 0.1794 | 0.2286 | 0.2406 | 0.2472 | 0.2500 |
| α | 0.0138 | 0.0188 | 0.0251 | 0.0267 | 0.0277 | 0.0284 |

The figure 3 is showing the value of stress for various load calculated at the long edge of the plate by the analytical formula and the case were also analyzed using numerical method with very fine mesh. The lower the element size the

more it came closer to the analytical one. The plate panel is 1800 mm in length and 500 mm in breadth with thickness of 8 mm and the pressure are 10 ton/ m².

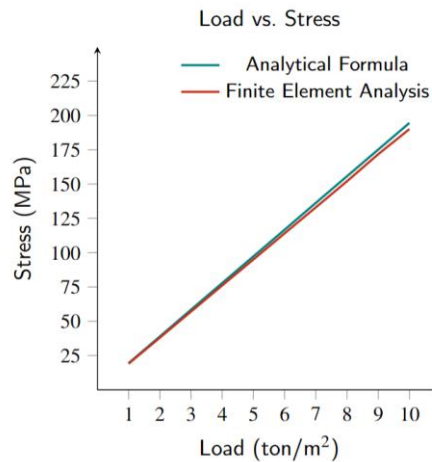


Fig. 3. Comparison of Stress obtained by Analytical formula and Numerical investigation as a function of Load.

Each plate supporting member can be assessed using the fixed-ended beam method under a uniformly distributed load (UDL). These formulas are closely aligned with the scantling formulas provided by the classification society for girders and stiffeners, differing primarily by the subtraction of certain coefficients

$$M_{\max} \text{ (at ends)} = w L^2 / 12$$

$$M_{\max} \text{ (at center)} = w L^2 / 24$$

The bending stress then become

$$\sigma = M_{\max} / Z$$

The deflection can also be checked by

$$\Delta_{\max} \text{ (at center)} = w L^4 / (384 E I)$$

In order to reduce the global bending moment of the structure it is a common practice to introduce pillars. Pillars usually provide simply support and reduce span length and that affect the bending moment. So, for estimating the parameter of the pillar the following method provided by Bureau Varitus (2024) can be used.

The maximum applied compressive axial load on a pillar to be determined by the following formula,

$$F_a = P b_{a-sup} \ell_{a-sup}$$

Finally, the contact pressure in N/mm^2 can be calculated by,

$$\sigma_c = F_a / A_c$$

If pillars are introduced to the structure there are good scopes to found such girder spans where both of the ends are simply supported and the girders got inter crossed each other like a grid. In such case simple grillage formula from W. Muckle is useful to consider. so, first it is required to calculate the reaction force at the intersection of the girders

$$W = (5/8) ((w_1 l_1 - w_2 l_2 (I_1 / I_2) (l_2^3 / l_1^3)) / (1 + (I_1 / I_2) (l_2^3 / l_1^3)))$$

It follows that the bending moment and deflection on two beams

$$M_1 = ((w_1 l_1 / 2) - (W / 2)) x - (w_1 x^2 / 2)$$

$$M_2 = ((w_2 l_2 / 2) + (W / 2)) y - (w_2 y^2 / 2)$$

$$\delta_1 = (5/384) (w_1 l_1^4 / EI_1) - (W l_1^3 / (48 EI_1))$$

$$\delta_2 = (5/384) (w_2 l_2^4 / EI_2) + (W l_2^3 / (48 EI_2))$$

These formulas specially plate and beam are unaffected by the overall bending moment generated by the entire structure's dimension. Regardless of the deck's length and breadth, the formulas remain consistent, making it insufficient to rely solely on dimensions derived from local strength analysis or scantling. Therefore, it's necessary to perform the global strength assessment using FEM or other empirical formulas.

3) Grillage analysis

When multiple stiffeners are present in each set, the complexity of solving the grillage problem increases. Instead of having a single concentrated reaction at each intersection, a series of unknown concentrated reactions must be considered. Consequently, there will be as many unknown forces as there are intersections between the stiffeners. This results in a set of linear simultaneous equations rather than a single simple equation.

The structural analysis conducted relies on the Navier method grillage theory. K Maneepan (2007) demonstrated that the Navier method provides an approximation close to the exact solution while being computationally more efficient compared to other grillage analysis methods. The Navier method

involves calculating deflections at the intersections of longitudinal girders (denoted by index g) and transverse beams (denoted by index b), a technique employed by several other researchers, including Clarkson (1965), Adam Sobey, VEDELER (1945), and J.I.R. Blake.

Initially, the basic properties of the girders are determined using the following approach: the flexural rigidity of the girder or stiffener, whether in the longitudinal or transverse direction, is calculated based on

$$D_g = \sum (E_{g(t)} I_{g(t)}) \text{ from } t=1 \text{ to } N_g$$

$$D_b = \sum (E_{b(i)} I_{b(i)}) \text{ from } i=1 \text{ to } N_b$$

The grillage analysis uses the Navier summations of points within the grillage to develop the deflection of the stiffeners. The values of the wave numbers m and n were kept at 17, as higher numbers extended computational time with only small increases in accuracy. The equation giving deflection of the stiffened plate, $w(x, y)$, is a double summation dependent on the wave numbers given as well as the length and breadth of the panel, L and B , and the longitudinal and transverse position along the panel, x and y , given below

The deflection, $w(x,y)$, at any point of the grillage is expressed by the following double summation of trigonometric series according to Navier's energy method Bedair (1997):

$$w(x, y) = \sum (\sum a_{(mn)} \sin((m \pi x) / L) \sin((n \pi y) / B))$$

from $m=1$ to ∞ and $n=1$ to ∞

The coefficient a_{mn} can be found from the following equation,

$$a_{mn} = (16 P L B) / (\pi^6 m n (m^4 (g + 1) (D_g / L^3) + n^4 (b + 1) (D_b / B^3)))$$

Hence, the complete expression for the deflection of the grillage can be found by substituting a_{mn} into a double sine series in Equation. The bending moment of the desired girder can be obtained by,

$$M_g = -D_g (\partial^2 w / \partial x^2)$$

$$M_b = -D_b (\partial^2 w / \partial x^2)$$

Finally, the direct stress in the axial direction on the girder cross section is given by the following expressions:

$$\sigma_{\max} = (E_{s(s)} M_s Z_s) / D_s$$

B. Numerical Approach

The initial shell elements are created using Rhinoceros 3D software, while Ansys Design Modeler is utilized to generate beam elements, reposition components, and specify the thicknesses of shell elements. Subsequently, all components are integrated into a unified part. The next step involves meshing the structure, which divides it into finite elements using two-node line elements and four-node shell elements. Shell elements are specifically used to model the deck plate and the girder webs

All stiffeners and face plates of the girders are modeled using beam elements. The aspect ratio of the shell elements is generally kept below 1.6, and the use of triangular shell elements is avoided. More than three elements are used along the height of the primary supporting members' webs. Once the pre-processing stage is complete, the load and boundary conditions are applied. All measurements are converted to millimeter unit.

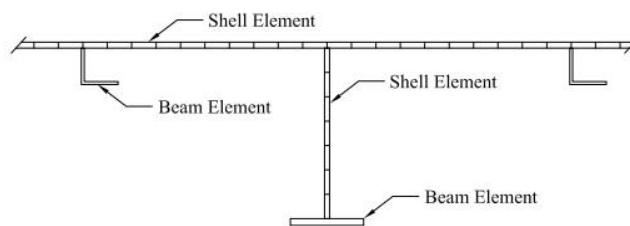


Fig. 4. Element selection for the structural members

Pressure 9.8066×10^{-2} MPa was applied to the deck plate, the edges were kept fixed as boundary condition.

At the post processing stage, plotting stress at the fixed end are avoided as it is excessively higher than other parts that generated as a byproduct of fixed support boundary condition. The negative bending moment at edge, reaction force,

resistance to deflection cause this phenomenon. Figure 5 shows the distance vs stress in long direction for normal and von mises stress. But, in reality the edges are not fixed and frames and brackets are connected to each girders and stiffeners that conduct the stress and distribute to the surrounding.

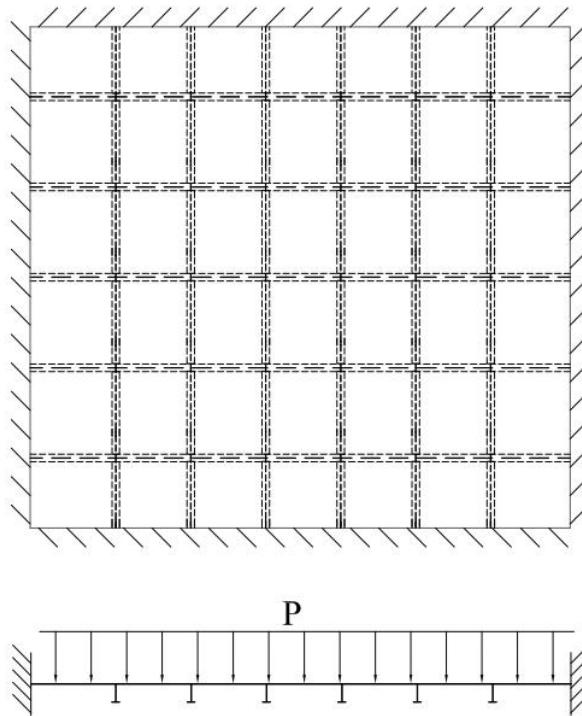


Fig. 5. Applied load and boundary condition of a typical Grillage

An efficient frame design can mitigate it but the tensile stress at the mid of the deck has no way to get transmitted unless additional supporting members are added. So, only stress at mid was considered for the structure.

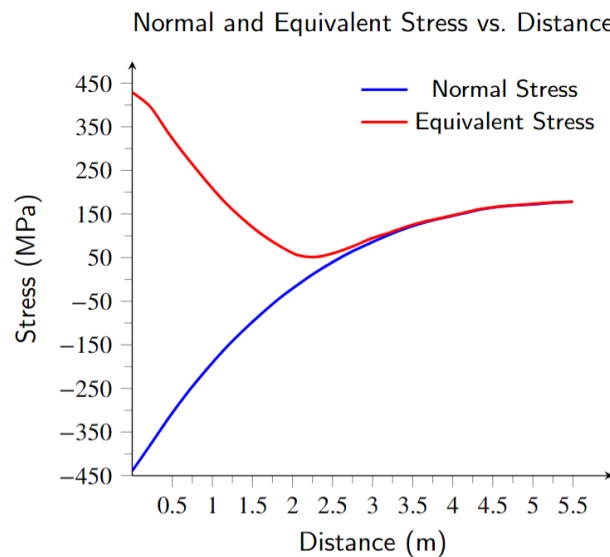


Fig. 6. Comparison of Normal Stress and Equivalent Stress as a function of Distance.

Figure.7 depicts a typical deck stress scenario for a pure grillage and girder supported structure. The deck plate got subjected to tensile stress at the end of plate panel and compressive stress at mid. The densely red marked area

indicated higher values for stress and the green stands for lower value while Figure 8 & 9 is showing the stress at the flange of the girders looking at bottom. At the bottom of the flange maximum stress was recorded for each case and at the bottom

of the web was also subjected to the stress plot but every time it was found lower than the flange. Up to a certain limit, usually stress greater than (+/-) 210 MPA the analysis is conducted repeatedly to make the structure lighter for each case. The

initial analysis has been conducted with the cases pure grillages only and after that stiffeners were introduced to transverse and both direction.

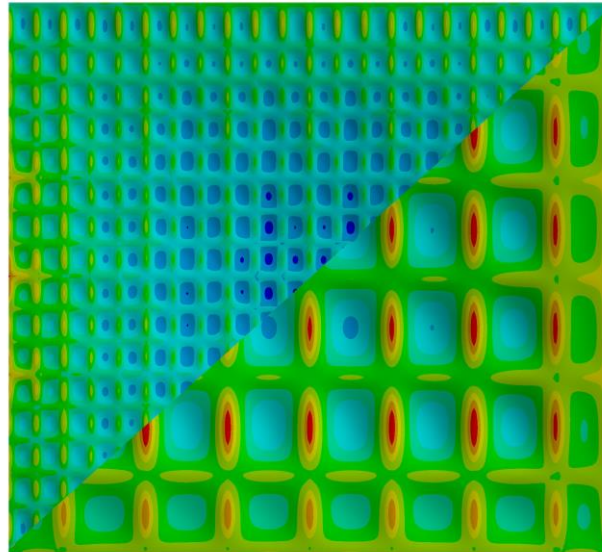


Fig. 7. Stress at deck

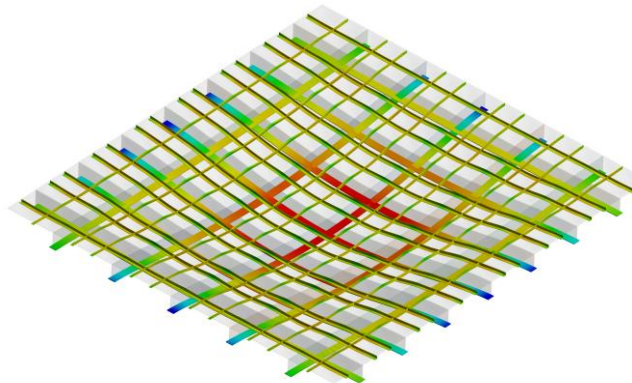


Fig. 8. Stress at beam elements

Table -3 Dimensions & Weight

| Title | Deck Plate | Girders | Web Frames | Stress | Weight |
|-------------------|------------|---------------------------------------|---------------------------------------|-----------|--------|
| Cases | t (mm) | T- $w_b \times t + flg \times t$ (mm) | T- $w_b \times t + flg \times t$ (mm) | flg (MPa) | (tons) |
| 1 | 7 | 450 X 14 + 200 X 15 | 450 X 14 + 200 X 15 | 218.69 | 16.87 |
| 2 | 9 | 450 X 12 + 175 X 14 | 450 X 12 + 175 X 14 | 226.29 | 16.91 |
| 3 | 18.5 | 450 X 14 + 200 X 15 | 450 X 14 + 200 X 15 | 209.07 | 25.1 |
| 4 | 17 | 200 X 14 + 450 X 14 | 200 X 14 + 450 X 14 | 210.52 | 24.14 |
| 5 | 15 | 175 X 16 + 350 X 14 | 175 X 16 + 350 X 14 | 221.75 | 22.44 |
| 6 | 12 | 250 X 12.5 + 175 X 14 | 250 X 12.5 + 175 X 14 | 216.5 | 19.07 |
| 7 | 8 | 125 X 9 + 250 X 8 | 125 X 9 + 250 X 8 | 222.46 | 15.89 |
| 1 _p | 6.5 | 450 X 8 + 100 X 8 | 450 X 8 + 100 X 8 | 216.4 | 12 |
| 1 _s | 8 | 450 X 6 + 200 X 15 | 250 X 6 + 175 X 14 | 221.28 | 12.2 |
| 1 _(-k) | 6.5 | 450 X 6 + 125 X 8 | 250 X 6 + 125 X 8 | 221.36 | 10.84 |

V. RESULTS

After investigating the cases the weight of each case have been measured and the percentage of area of every structural member were recorded that contributed to the total weight. It is found that for most of the cases the deck plate contributes to the maximum area of the structure which is about 41% to 57% depending on the cases and little increment on plate thickness affects the whole weight of the structure (each additional millimeter can increase the weight by approximately 0.86335 tons) and where the spacing was greater more thick plate was needed for local strength so the case where the spacing got lesser made the structure more lighter. The case 7 and 1 have the least and equal equal spacing and plate panel. Though the

case 7 stands as an optimized structure, a ship usually don't has this much frames to support that huge number of girders at both direction and welding of such number of structures within limited space also create difficulties and thereby case 1 is winner that save 33% weight comparing to case 3 which is the worst case.

From all of the cases with equal longitudinal and trans-verse girders the case 1 is the best material saving and practical combination case. Case 2 was closer to it but the length of the long edge of the plate panel is almost double and subjected to more local pressure and required little bit of extra thickness that increased 5% of the weight

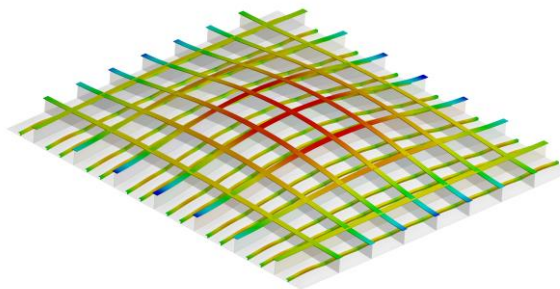


Fig. 9. Stress at Flanges

After the initial trial of the grillage structure the effective one (case 1) has been converted to girder supported one. The center girder was made deeper to bear the moment, the side girder were made smaller and the transverse girders were the smallest. Though the plate experienced bit more stress at center line and required extra thickness, other supporting members' got light and reduced overall weight and defeated case 1 in terms of material saving that reduced 6

Finally pillars were added (6 pillars for grillage and 5 pillars for girder supported system under the inter section of frames at almost equal distance, 3 m height each, Size: DN80, thk 8) for the mentioned cases and it reduced the moment as the pillar works as simply support, it reduced the span length. It affected the structure significantly, the section modulus requirement got lower and the deck plate thickness also got reduced. After re analyzing the cases again, finally the most optimized one is the girder supported structure with pillars. The table 3 shows the outcomes of the experiments.

VI. CONCLUSION

Structures with stiffeners are more effective at resisting local deformations in plates, allowing them to be thinner while maintaining strength, thus reducing weight. In contrast, grillage structures distribute loads evenly across all members, requiring each member to be of uniform dimensions, which increases the overall weight. Girder-supported structures, on the other hand,

allow for lighter dimensions of other members. In both scenarios, pillars are vital for weight reduction, and local structural analyses or scantling are particularly beneficial when pillars are included.

While pillars can contribute to weight reduction in a structure, their use can be problematic in certain scenarios. Incorporating pillars requires space beneath the deck, which is impractical for ferries transporting cars or passenger vessels that need clear areas for movement, large conference rooms, or machinery requiring unobstructed space. In these cases, a grillage structure is preferable. Conversely, for ships carrying heavy loads on the main deck where the under-deck space is unoccupied, pillars can be utilized without limitations, allowing for adjustments to the girder-supported structure based on stress distribution. In such scenarios, a grillage structure would be costly.

Thus, the research demonstrates that grillage structures are less material-efficient than girder-supported structures under unrestricted conditions.

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